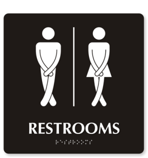
NAME Key

LINEAR ALGEBRA/MATH 270
SHANNON MYERS

PLEASE MAKE SURE YOU ARE TAKING THE CORRECT EXAM FOR YOUR CLASS!!!

- π 100 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π SCIENTIFIC CALCULATOR ONLY
- π NO NOTES, AND/OR FORMULAS PERMITTED
- π PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED



ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED…THIS MEANS NO BATHROOM BREAKS!

MATH 270/Myers NAME Key

EXAM 1/Chapter 1-2.2 OF THE ELEMENTARY LINEAR ALGEBRA WORKBOOK

100 POINTS POSSIBLE

YOUR WORK MUST BE ORGANIZED AND CLEAR

SCIENTIFIC CALCULATOR ONLY

1. (6 POINTS) Find the solution set of the system of equations represented in the coefficient matrix of a homogeneous system of equations. If applicable, please parameterize.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

augmented matrix

$x_1 + x_3 = 0 \rightarrow x_1 = x_3 = 0$

$x_2 - 5x_4 = 0 \rightarrow x_2 = 5x_4 = 5t$

$x_3 = 0 \rightarrow x_3 = 0$

$x_4 = t$

$$\boxed{\{(0, 5t, 0, t) : t \in \mathbb{R}\}}$$

2. (10 POINTS) Please circle true or false.

a. T F A homogeneous system of equations only has the trivial solution.

b. T F Multiplication of matrices is commutative.

c. T F A set $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}, k \geq 2$ is linearly independent if and only if at least one of the vectors $\mathbf{v}_i$ can be written as a linear combination of the other vectors in S.

skip bad problem

d. T F The transpose of the product of two matrices of order n is equal to the product of the transposes.

e. T F There is only one way to parametrically represent the solution set of a linear equation.

3. (8 POINTS) Prove the following distributive property for vectors in R^n: $(c+d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$.

Pf: Let $\vec{u} = (u_1, u_2, \dots, u_n) \ni u_i \in \mathbb{R}$ for $i = 1, 2, \dots, n$. Let $c, d \in \mathbb{R}$.

$(c+d)\vec{u} = (c+d)(u_1, u_2, \dots, u_n)$

$= ((c+d)u_1, (c+d)u_2, \dots, (c+d)u_n)$ defn of vector scalar mult

$= (cu_1 + du_1, cu_2 + du_2, \dots, cu_n + du_n)$ $\mathbb{R}$ is dist. over addition

$= (cu_1, cu_2, \dots, cu_n) + (du_1, du_2, \dots, du_n)$ defn of vector addition

$= c\vec{u} + d\vec{u}$. //

4. (10 POINTS) Determine whether the set $W = \{(s, t, 3s-4t) : s, t \text{ are real numbers}\}$ is a subspace of R^3 with the standard operations.

W is a nonempty subset of $\mathbb{R}^3$, since $(0, 0, 3\cdot 0 - 4\cdot 0) = (0,0,0) \in W$ and $\mathbb{R}^3$.

Let $\vec{u} = (u_1, u_2, 3u_1 - 4u_2)$, $\vec{v} = (v_1, v_2, 3v_1 - 4v_2) \ni u_i, v_i \in \mathbb{R}$ for $i = 1, 2$. Let $c \in \mathbb{R}$.

closure under +

$\vec{u} + \vec{v} = (u_1, u_2, 3u_1 - 4u_2) + (v_1, v_2, 3v_1 - 4v_2)$

$= (u_1 + v_1, u_2 + v_2, (3u_1 - 4u_2) + (3v_1 - 4v_2))$ defn of vector +

$= (u_1 + v_1, u_2 + v_2, (3u_1 + 3v_1) + (-4u_2 - 4v_2))$ $\mathbb{R}$ is commutative

$= (u_1 + v_1, u_2 + v_2, 3(u_1 + v_1) - 4(u_2 + v_2))$ $\mathbb{R}$ is distributive

$\in W$. ✓

closure under scalar mult

$c\vec{u} = c(u_1, u_2, 3u_1 - 4u_2)$

$= (cu_1, cu_2, c(3u_1 - 4u_2))$ defn of vector scalar mult

$= (cu_1, cu_2, c(3u_1) + c(-4u_2))$ $\mathbb{R}$ is dist.

$= (cu_1, cu_2, 3(cu_1) - 4(cu_2))$ $\mathbb{R}$ is comm.

$\in W$ ✓

$\boxed{\text{W is a subspace of } \mathbb{R}^3}$

5. (16 POINTS) Consider the following matrices A, B, and C.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \\ -6 & 9 & 7 \end{bmatrix} \qquad B^T = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 13 & 1 \\ 1 & -1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -1 \\ 5 \\ 10 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 13 & -1 \\ 3 & 1 & 0 \end{bmatrix}$$

a. (4 POINTS) Find $A - 3B$.

$$= \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \\ -6 & 9 & 7 \end{bmatrix} + \begin{bmatrix} -6 & 0 & -3 \\ 3 & -39 & 3 \\ -9 & -3 & 0 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -5 & 3 & -1 \\ 7 & -39 & 4 \\ -15 & 6 & 7 \end{bmatrix}}$$

b. (5 POINTS) Find AC.

$$= \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \\ -6 & 9 & 7 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 15 + 20 \\ -4 + 0 + 10 \\ 6 + 45 + 70 \end{bmatrix} \rightarrow = \boxed{\begin{bmatrix} 34 \\ 6 \\ 121 \end{bmatrix}}$$

c. (7 POINTS) Find $(AB)^T$.

$$(AB)^T = B^T A^T$$

$$= \begin{bmatrix} 2 & -1 & 3 \\ 0 & 13 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & -6 \\ 3 & 0 & 9 \\ 2 & 1 & 7 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 5 & 11 & 0 \\ 41 & 1 & 124 \\ -2 & 4 & -15 \end{bmatrix}}$$

6. (8 POINTS) Determine the polynomial function whose graph passes through the points (2, 5), (3, 2), (4, 5).

$p(x) = a_0 + a_1 x + a_2 x^2$

$p(2) = 5 = a_0 + 2a_1 + 4a_2$

$p(3) = 2 = a_0 + 3a_1 + 9a_2$

$p(4) = 5 = a_0 + 4a_1 + 16a_2$

$$\begin{aligned} a_0 + 2a_1 + 4a_2 &= 5 \\ a_0 + 3a_1 + 9a_2 &= 2 \\ a_0 + 4a_1 + 16a_2 &= 5 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 1 & 3 & 9 & 2 \\ 1 & 4 & 16 & 5 \end{array}\right]$$

$-R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 1 & 4 & 16 & 5 \end{array}\right]$$

$-R_1 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 2 & 12 & 0 \end{array}\right]$$

$-2R2 + R3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 2 & 6 \end{array}\right]$$

$a_0 + 2a_1 + 4a_2 = 5$

$a_1 + 5a_2 = -3$

$2a_2 = 6 \rightarrow a_2 = 3$

$a_1 + 5(3) = -3 \rightarrow a_1 = -18$

$a_0 + 2(-18) + 4(3) = 5$

$a_0 = 29$

$\boxed{p(x) = 29 - 18x + 3x^2}$

7. (8 POINTS) Show that the set, together with the indicated operations, is not a vector space by identifying one of the ten vector space axioms that fail.
$W = \{(x, y) : x, y \in R, x \geq 0\}$ with the standard operations in R^2.

Let $(x,y) = (1,2) \in W$, and let $c = -3$

$c(x,y) = -3(1,2) = (-3,-6) \notin W$ since -3 isn't positive.

8. (8 POINTS) Please solve the linear system by hand using elementary row operations.

$$\begin{aligned} -x + y + 2z &= 1 \\ 2x + 3y + z &= -2 \\ 5x + 4y + 2z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ 5 & 4 & 2 & 4 \end{array}\right]$$

$\downarrow$

$2R_1 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 5 & 5 & 0 \\ 5 & 4 & 2 & 4 \end{array}\right]$$

$\downarrow$

$5R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 5 & 5 & 0 \\ 0 & 9 & 12 & 9 \end{array}\right]$$

$\downarrow$

$-9R_2 + 5R_3 \to R_3$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 5 & 5 & 0 \\ 0 & 0 & 15 & 45 \end{array}\right]$$

$\downarrow$

$\frac{1}{15}R_3 \to R_3, \ \frac{1}{5}R_2 \to R_2$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$-R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$-2R_3 + R_1 \to R_1$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\downarrow$

$-R_2 + R_1 \to R_1$

$$\left[\begin{array}{ccc|c} -1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\downarrow$

$-1R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\boxed{\{(2,-3,3)\} \text{ consistent system with independent equations}}$$

9. (8 POINTS) Let $\mathbf{v}_1 = (10,-5)$, $\mathbf{v}_2 = (7,-2)$, and $\mathbf{u} = (-21,11)$. If possible, please write $\mathbf{u}$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{u}$$

$$c_1(10,-5) + c_2(7,-2) = (-21,11)$$

$$10c_1 + 7c_2 = -21$$
$$-5c_1 - 2c_2 = 11$$

$$\left[\begin{array}{cc|c} 10 & 7 & -21 \\ -5 & -2 & 11 \end{array}\right]$$

$\downarrow$ $R_1 + 2R_2 \to R_2$

$$\left[\begin{array}{cc|c} 10 & 7 & -21 \\ 0 & 3 & 1 \end{array}\right] \to \begin{array}{l} 10c_1 + 7c_2 = -21 \to c_1 = -21 - 7\left(\frac{1}{3}\right) = \frac{-63-7}{10\cdot 3} = -\frac{7}{3} \\ 3c_2 = 1 \to c_2 = \frac{1}{3} \end{array}$$

$$\boxed{-\frac{7}{3}(10,-5) + \frac{1}{3}(7,-2) = (-21,11)}$$

$$\boxed{c_1 = -\frac{7}{3},\ c_2 = \frac{1}{3}}$$

10. (8 POINTS) Determine whether $S = \{(-2,5),(6,-1)\}$ is a basis for R^2.

Spanning: Let $\vec{u} = (u_1,u_2) \ni u_1, u_2 \in \mathbb{R}$

$$c_1(-2,5) + c_2(6,-1) = (u_1,u_2)$$
$$-2c_1 + 6c_2 = u_1$$
$$5c_1 - c_2 = u_2$$

$$\left[\begin{array}{cc|c} -2 & 6 & u_1 \\ 5 & -1 & u_2 \end{array}\right] \xrightarrow{5R_1 + 2R_2 \to R_2} \left[\begin{array}{cc|c} -2 & 6 & u_1 \\ 0 & 28 & 5u_1 + 2u_2 \end{array}\right]$$

$-28R_1 + 6R_2 \to R_1$

$$\left[\begin{array}{cc|c} -2 & 0 & -28u_1 + 6(5u_1+2u_2) \\ 0 & 28 & 5u_1+2u_2 \end{array}\right] \to \begin{array}{l} -2c_1 = 2u_1 + 12u_2 \to c_1 = -u_1 - 6u_2 \\ 28c_2 = 5u_1 + 2u_2 \to c_2 = \frac{5}{28}u_1 + \frac{1}{14}u_2 \end{array}$$

So S spans R^2 ✓

Linear Independence:

$$c_1(-2,5) + c_2(6,-1) = (0,0)$$
$$-2c_1 + 6c_2 = 0 \to -2c_1 + 6(5c_1) = 0 \to 28c_1 = 0 \to c_1 = 0$$
$$5c_1 - c_2 = 0 \to c_2 = 5c_1 \to c_2 = 5(0) = 0$$

Since $c_1 = c_2 = 0$, S is linearly independent. ✓

$$\boxed{S \text{ is a basis for } R_2}$$

11. (10 POINTS) A college dormitory houses 300 students. Those who watch an hour or more of television on any day always watch for less than an hour the next day. One-fourth of those who watch television for less than an hour one day will watch an hour or more the next day. Half of the students watched television for an hour or more today.

a. Please find the matrix of transition.

$$P = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \geq 1\text{hr} & < 1\text{hr} \\ \left[\begin{array}{c} 0 \\ 1 \end{array}\right. & \left.\begin{array}{c} 1/4 \\ 3/4 \end{array}\right] \end{array}\begin{array}{l} \\ \geq 1\text{ hr} \\ < 1\text{hr} \end{array} \qquad X_0 = \begin{bmatrix} 150 \\ 150 \end{bmatrix}\begin{array}{l} \geq 1\text{hr} \\ < 1\text{hr} \end{array}$$

b. How many will watch television for an hour or more tomorrow?

$$PX_0 = \begin{bmatrix} 0 & 1/4 \\ 1 & 3/4 \end{bmatrix}\begin{bmatrix} 150 \\ 150 \end{bmatrix} = \begin{bmatrix} 37.5 \\ 262.5 \end{bmatrix} = X_1$$

$$\boxed{\text{Approximately 38 students will watch television for an hour or more tomorrow.}}$$

c. How many will watch television for an hour or more in two days?

$$P(X_1) = \begin{bmatrix} 0 & 1/4 \\ 1 & 3/4 \end{bmatrix}\begin{bmatrix} 37.5 \\ 262.5 \end{bmatrix} = \begin{bmatrix} 65.6 \\ 234.4 \end{bmatrix}$$

$$\boxed{\text{Approximately 66 students will watch television for an hour or more in two days.}}$$